ON SOME PERIODIC ORBITS OF THE FOURTH–ORDER DIFFERENTIAL EQUATION $u'''' + qu'' + pu = \varepsilon F(u, u', u'', u''')$

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Abstract. We provide sufficient conditions for the existence of periodic solutions of the fourth–order differential equation

$$u'''' + qu'' + pu = \varepsilon F(u, u', u'', u'''),$$

where q, p and ε are real parameters, ε is small and F is a nonlinear function.

1. Introduction and statement of the main results

The goal of this paper is to study the periodic solutions of the fourth–order differential equation

$$u'''' + qu'' + pu = \varepsilon F(u, u', u'', u''') \quad \text{with} \quad p > 0, \tag{1}$$

where q, p and ε are real parameters, ε is small and F is a nonlinear function. The prime denotes derivative with respect to an independent variable l.

In general to obtain analytically periodic solutions of a differential system is a very difficult problem, usually impossible. Here using the averaging theory this difficult problem for the differential equations (1) is reduced to find the zeros of a nonlinear system of three or two function. We must mention that the averaging theory for finding periodic solutions in general does not provide all the periodic solutions of the system. For more information and details about the averaging theory see section 2 and the references quoted there.

Equations (1) appear in many places. For instance, Champneys [7] analyzes a class of equations (1) looking mainly for homoclinic orbits.

When $F = \pm u^2$ equation (1) can come from the description of the travelling–wave solutions of the Korteweg–de Vries equation with an additional fifth-order dispersive term. Extended fifth–order Korteweg–de Vries equations have been considered in [5, 8, 9, 15, 16, 19]. This equation has been used to describe chains of coupled nonlinear oscillators [20] and most notably gravity–capillary shallow water waves [3, 13, 24]. For other derivation of (1) see [11, 12].

Another nonlinearity is $F = \pm u^3$, then equation (1) is called the *Extended Fischer–Kolmogorov* equation or the *Swift-Hohenberg* equation see [4, 14], and in other places, see for instance the book [21] and [1, 6].

Some results on the periodic orbits for extended Fisher–Kolmogorov and Swift–Hohenberg equations of the form

$$u'''' + qu'' + \alpha(l)u = f(l, u, u', u''), \tag{2}$$

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