PHASE PORTRAITS OF THE RICCATI QUADRATIC POLYNOMIAL DIFFERENTIAL SYSTEMS

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ABSTRACT. In this paper we characterize the phase portrait of the Riccati quadratic polynomial differential systems

$$\dot{x} = \alpha_2(x), \quad \dot{y} = ky^2 + \beta_1(x)y + \gamma_2(x),$$

with $(x, y) \in \mathbb{R}^2$, $\gamma_2(x)$ non-zero (otherwise the system is a Bernoulli differential system), $k \neq 0$ (otherwise the system is a Lienard differential system), $\beta_1(x)$ a polynomial of degree at most 1, $\alpha_2(x)$ and $\gamma_2(x)$ polynomials of degree at most 2, and the maximum of the degrees of $\alpha_2(x)$ and $ky^2 + \beta_1(x)y + \gamma_2(x)$ is 2. We give the complete description of their phase portraits in the Poincaré disk (i.e. in the compactification of $\mathbb{R}^2$ adding the circle $\mathbb{S}^1$ of the infinity) modulo topological equivalence.

1. INTRODUCTION AND STATEMENT OF THE MAIN RESULTS

Numerous problems of applied mathematics are modeled by quadratic polynomial differential systems, see for instance [9]. Excluding linear systems, such systems are the ones with the lowest degree of complexity, and the large bibliography on the subject proves its relevance. We refer for example to the books of Ye Yanqian et al. [12], Reyn [10], and Artes, Llibre, Schlomiuk, Vulpe [1], and the surveys of Coppel [3], and Chicone and Jinghuang [2] are excellent introductory readings to the quadratic polynomial differential systems.