PERIODIC SOLUTIONS OF CONTINUOUS THIRD–ORDER DIFFERENTIAL EQUATIONS WITH PIECEWISE POLYNOMIAL NONLINEARITIES

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ABSTRACT. We consider third–order autonomous continuous piecewise differential equations in the variable x. For such differential equations with nonlinearities of the form x^m, we investigate their periodic solutions using the averaging theory.

1. Introduction and Statement of the Main Result

For studying some electrical circuits Sprott and Sun [9, 10, 11] considered the third order differential equation $\dddot{x} = -\dot{x} - a\ddot{x} + g(x)$, where g is an elemental piecewise function. He showed that some of these equations exhibit chaos.

In this paper we are interested in studying the third order of differential equations of the form

$$\dddot{x} = -\dot{x} + \varepsilon|\ddot{x}| - \varepsilon a x^m, \tag{1}$$

where a and ε are parameters and ε is small. But our interest is in studying how their periodic solutions depend on the parameter a and on the exponent m.

We can write the third order differential equations (1) as the following differential system of first order

$$\begin{aligned} \dot{x} &= y, \\ \dot{y} &= z, \\ \dot{z} &= -y + \varepsilon|z| - \varepsilon a x^m, \end{aligned} \tag{2}$$

where the dot denotes derivative with respect an independent variable t, usually the time.

We remark that the differential system (2) is only continuous due to the existence of the term $|z|$, so we cannot apply to it the classical averaging

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