CENTRAL CONFIGURATIONS OF THE CIRCULAR RESTRICTED 4-BODY PROBLEM WITH THREE EQUAL PRIMARIES IN THE COLLINEAR CENTRAL CONFIGURATION OF THE 3-BODY PROBLEM

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Abstract. In this paper we classify the central configurations of the circular restricted 4-body problem with three primaries with equal masses at the collinear configuration of the 3-body problem and an infinitisimal mass.

1. Introduction and results

The well-known Newtonian n-body problem concerns with the motion of n mass points with positive mass m_i moving under their mutual attraction in $\mathbb{R}^d$ in accordance with Newton's law of gravitation.

The equations of the motion of the n-body problem are :

$$\ddot{r}_i = -\sum_{j=1, j\neq i}^{n} \frac{m_j(r_i - r_j)}{r_{ij}^3}, \quad 1 \leq i \leq n,$$

where we have taken the unit of time in such a way that the Newtonian gravitational constant be one, and $r_i \in \mathbb{R}^d$ $(i = 1, \dots, n)$ denotes the position vector of the i-body, $r_{ij} = |r_i - r_j|$ is the Euclidean distance between the i-body and the j-body.

The solutions of the 2-body problem (also called the Kepler problem) has been completely solved. Unfortunately the solutions for the n-body for $n > 2$ is still an open problem.

For the Newtonian n-body problem the simplest possible motions are such that the configuration formed by the n-bodies is constant up to rotations and scaling, such motions are called the *homographic solutions* of the n-body problem, and are the unique known explicit solutions of the n-body problem when $n > 2$. Only some special configurations of particles are allowed in the homographic solutions of the n-body problem, called by Wintner [64] *central configurations*. Also, central configurations are of utmost importance when studying bifurcations of the hypersurfaces of constant energy and angular momentum, for more details see Meyer [47] and Smale [60].

More precisely, let

$$M = m_1 + \cdots + m_n, \quad c = \frac{m_1 r_1 + \cdots + m_n r_n}{M},$$

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