LIMIT CYCLES OF PLANAR CONTINUOUS PIECEWISE DIFFERENTIAL SYSTEMS SEPARATED BY A PARABOLA AND FORMED BY AN ARBITRARY LINEAR AND QUADRATIC CENTERS

JAUME LLIBRE

Abstract. Due to their applications to many physical phenomena during these last decades the interest for studying the continuous or discontinuous piecewise differential systems has increased strongly. The limit cycles play a main role in the study of any planar differential system. Up to now the major part of papers which study the limit cycles of the planar piecewise differential systems have considered systems formed by two pieces separated by one straight line. Here we consider planar continuous piecewise differential systems separated by a parabola.

We prove that the planar continuous piecewise differential systems separated by a parabola and formed by a linear center and a quadratic center have at most one limit cycle. Moreover there are systems in this class exhibiting one limit cycle. So in particular we have solved the extension of the 16th Hilbert problem to this class of differential systems.

1. Introduction and results

Andronov, Vitt and Khaikin [1] started in a serious way the study of the piecewise differential systems mainly motivated for their applications to some mechanical systems, and now these systems still continue to receive the attention of many researchers. Recently these differential systems are widely used to model processes appearing in mechanics, electronics, economy, etc., see for instance the books [3] and [19], and the survey [17], as well as the hundreds of references cited there.

While the more studied piecewise differential systems are the discontinuous ones, here we will deal with a class of the continuous piecewise differential systems in the plane.

The simplest possible continuous but nonsmooth piecewise differential systems are the ones having only two pieces formed by two linear differential systems separated by a straight line in the plane $\mathbb{R}^2$. Thus in 1990 Lum and Chua conjectured in [15, 16] that such continuous piecewise linear differential systems have at most one limit cycle. In 1998 this conjecture was proved by Freire et al. [6]. In 2013 a new and shorter proof was done by Llibre, Ordóñez and E. Ponce [12], and recently another proof has been provided by Carmona, Fernández-Sánchez and Novaes [4]. But in all these papers the authors forgot to analyze the case when the two linear differential systems which form the piecewise linear differential system have no equilibrium points, this case was studied in Llibre and Teixeira [13] in 2016 where it is proved that such continuous piecewise systems have no limit cycles.

Let $p(x, y) = 0$ be a parabola. A *continuous piecewise differential system in the plane with two pieces separated by a parabola* is a differential system of the form

$$\dot{x} = f_+(x, y), \quad \dot{y} = g_+(x, y), \quad \text{in the region } p(x, y) \geq 0;$$

and

$$\dot{x} = f_-(x, y), \quad \dot{y} = g_-(x, y), \quad \text{in the region } p(x, y) \leq 0;$$

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