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The cyclicity of the period annulus of a reversible quadratic system

Changjian Liu
School of Mathematics (Zhuhai), Sun Yat-sen University, Zhuhai 519086, China
(liuchangj@mail.sysu.edu.cn)

Chengzhi Li
School of Mathematical Sciences, Peking University, Beijing 100871, China
(licz@math.pku.edu.cn)

Jaume Llibre
Departament de Matemàtiques, Universitat Autònoma de Barcelona, 08193 Bellaterra, Barcelona, Catalonia, Spain
(jllibre@mat.uab.cat)

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We prove that perturbing the periodic annulus of the reversible quadratic polynomial differential system $\dot{x} = y + ax^2$, $\dot{y} = -x$ with $a \neq 0$ inside the class of all quadratic polynomial differential systems we can obtain at most two limit cycles, including their multiplicities. Since the first integral of the unperturbed system contains an exponential function, the traditional methods cannot be applied, except in Figuerasa, Tucker and Villadelprat (2013, *J. Diff. Equ.*, **254**, 3647–3663) a computer-assisted method was used. In this paper, we provide a method for studying the problem. This is also the first purely mathematical proof of the conjecture formulated by Dumortier and Roussarie (2009, *Discrete Contin. Dyn. Syst.*, **2**, 723–781) for $q \leqslant 2$. The method may be used in other problems.

Keywords: Perturbation of quadratic reversible centre; Abelian integral; limit cycle

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1. Introduction and statement of the main results

We recall that a *centre* of a planar differential system is a singular point p of the system having a neighbourhood filled up of periodic orbits with the unique exception of the point p. The *period annulus* of a centre is the maximal region filled up with the periodic orbits surrounding the centre.

There is a big programme whose objective is to find the exact upper bound for the number of limit cycles that can bifurcate from the periodic orbits of the period