PHASE PORTRAITS OF SEPARABLE QUADRATIC SYSTEMS AND A BIBLIOGRAPHICAL SURVEY ON QUADRATIC SYSTEMS

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Abstract. Although planar quadratic differential systems and their applications have been studied in more than one thousand papers, we still have no complete understanding of these systems. In this paper we have two objectives.

First we provide a brief bibliographical survey on the main results about quadratic systems. Here we do not consider the applications of these systems to many areas as in Physics, Chemist, Economics, Biology, ...

Second we characterize the new class of planar separable quadratic polynomial differential systems. For such class of systems we provide the normal forms which contain one parameter, and using the Poincaré compactification and the blow up technique, we prove that there exist 10 non-equivalent topological phase portraits in the Poincaré disc for the separable quadratic polynomial differential systems.

1. Introduction and statement of the main result

Let $P(x, y)$ and $Q(x, y)$ be two real polynomials of degree 2. Then the differential system

$$\dot{x} = P(x, y), \qquad \dot{y} = Q(x, y), \tag{1}$$

is called a *planar quadratic polynomial differential system*, or in what follows simply a *quadratic system*. As usual the dot denotes derivative with respect to an independent variable t, called the time.

Quadratic systems began to be studied at the beginning of the twentieth century. According to Coppel [76] it seems that the first work on quadratic systems was written in 1904 by Büchel [41]. In 1966 Coppel [76] published a short survey on quadratic systems, another short survey on these systems appeared in 1982 by Chicone and Tian [63].

Quadratic systems have been intensively studied in the past several decades and a large number of valuable results were obtained, see the books [22, 189, 226] dedicated completely to quadratic systems and references therein, and

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