THE CYCLICITY OF THE PERIOD ANNULUS OF A REVERSIBLE QUADRATIC SYSTEM

CHENGZHI LI[1] AND JAUME LLIBRE[2]

ABSTRACT. We prove that perturbing the periodic annulus of the reversible quadratic polynomial differential system $\dot{x} = y + ax^2$, $\dot{y} = -x$ with $a \neq 0$ inside the class of all quadratic polynomial differential systems we can obtain at most one limit cycle. Since the first integral of the unperturbed system contains an exponential function, we have to use some special method to finish the study.

1. INTRODUCTION AND STATEMENT OF THE MAIN RESULTS

We recall that a *center* of a planar differential system is a singular point p of the system having a neighborhood filled up of periodic orbits with the unique exception of the point p. The *period annulus* of a center is the maximal region filled up with the periodic orbits surrounding the center.

There is a big program whose objective is to find the exact upper bound for the number of limit cycles which can bifurcate from the periodic orbits of the period annuli of the quadratic polynomial differential systems under quadratic perturbations, see for instance the second part of the book of Christhopher and Li [3]. This upper bound is called *the cyclicity of the period annulus*. This program started with Arnold [1, 2] and has produced more than one hundred articles, see for instance the references of [3].

Here we contribute to this program determining this upper bound for the period annuli of the centers of the quadratic polynomial differential systems

$$\dot{X} = Y + aX^2, \qquad \dot{Y} = -X, \tag{1}$$

with $a \neq 0$. We note that to study the cyclicity of the period annulus of a system (1) is equivalent to study the cyclicity of the period annulus

2010 *Mathematics Subject Classification.* Primary 34C07, 34C08, 37G15.
Key words and phrases. Quadratic center, reversible center, limit cycle, Liénard system.