ON THE NUMBER OF LIMIT CYCLES IN PLANAR DISCONTINUOUS PIECEWISE LINEAR DIFFERENTIAL SYSTEMS OF SADDLE-NODE TYPE

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ABSTRACT. This paper deals with planar discontinuous piecewise linear differential systems with two zones separated by a straight line. Using the Liénard-like canonical form, we investigated the maximum number of limit cycles of saddle-node type.

1. Introduction and statement of the main results

Planar piecewise linear differential systems attract both mathematician's and engineer's attention in recent years because such systems are widely used to describe many real phenomena and different modern devices, see more details for [1]. It is obvious that class of piecewise linear differential system with two zones separated by a straight line is the simplest class of these system as follows.

$$\begin{pmatrix}\dot{x}\\ \dot{y}\end{pmatrix}=\begin{cases}\begin{pmatrix}a_{1,1}^- & a_{1,2}^-\\ a_{2,1}^- & a_{2,2}^-\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}+\begin{pmatrix}b_1^-\\ b_2^-\end{pmatrix} & \text{if } x<0,\\ \begin{pmatrix}a_{1,1}^+ & a_{1,2}^+\\ a_{2,1}^+ & a_{2,2}^+\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}+\begin{pmatrix}b_1^+\\ b_2^+\end{pmatrix} & \text{if } x>0,\end{cases} \tag{1}$$

where the dot denote the derivative with respect to t.

Following Filippov's convention, we can distinguished the separating line $x=0$ into three open regions

$$\begin{aligned}\Sigma_c &= \{(0,y)|(a_{1,2}^-y+b_1^-)(a_{1,2}^+y+b_1^+)>0\},\\ \Sigma_a &= \{(0,y)|(a_{1,2}^-y+b_1^-)>0,(a_{1,2}^+y+b_1^+)<0\},\\ \Sigma_r &= \{(0,y)|(a_{1,2}^-y+b_1^-)<0,(a_{1,2}^+y+b_1^+)>0\}.\end{aligned} \tag{2}$$

The sets Σ_c and $\Sigma_a\cup\Sigma_r$ are called the *crossing sets* and *sliding sets* of system (1), respectively. If a isolated periodic orbit of system (1) has sliding points, then it will be called *sliding limit cycle*. Otherwise we speak of *crossing limit cycle*.

In 1990 Lum and Chua [15] conjectured that a continuous piecewise linear differential system (1) has at most one limit cycle. In 1998 Freire et al [4] gave a positive answer for this conjecture by careful qualitative analysis, for a shorter proof see [12]. Note that continuous piecewise linear differential system (1) cannot have sliding segment, thus its has no sliding limit cycle.

2010 *Mathematics Subject Classification.* 34A36, 34C07, 37G15.
Key words and phrases. Limit cycle, Discontinuous linear systems.