ON THE EXISTENCE AND UNIQUENESS OF LIMIT CYCLES FOR CONTINUOUS PIECEWISE-LINEAR LIÉNARD DIFFERENTIAL SYSTEMS WITH THREE ZONES

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Abstract. This paper deals with planar continuous piecewise linear Liénard differential systems with three zones separated by two vertical lines without symmetry. We show the existence and uniqueness of limit cycles for systems with a unique singular point located in the middle zone. As an application we investigate the existence of canard limit cycles for a class of continuous piecewise linear slow-fast Liénard differential systems.

1. Introduction and statement of the main results

One of the most important problems in the qualitative theory of planar differential systems is the determination and distribution of their limit cycles. This question, focus on planar polynomial systems, is known as the second part of Hilbert's sixteenth problem, see for instance [13]. Since Hilbert's sixteenth problem turns out to be a strongly difficult one, Smale [29] restricted it to the Liénard differential systems

$$\frac{dx}{dt} = F(x) - y, \qquad \frac{dy}{dt} = G(x), \tag{1}$$

where $F(x)$ and $G(x)$ are polynomial functions in the variable x of degree n and m respectively. If $m = 1$, then systems (1) are called classical Liénard differential systems. In 1977 Lins, Melo and Pugh [15] conjectured that the classical Liénard differential systems have at most $[(N-1)/2]$ limit cycles. This conjecture holds for $n = 1, 2, 3, 4$, and it does not holds for $n \geqslant 6$. Up to now it is still open for $n = 5$, see the survey paper [23] and references therein. For the generalized Liénard differential systems, see [10, 16, 30] for lower bounds on the number of limit cycles.

Piecewise linear Liénard differential systems are very important within the realm of nonlinear dynamical systems. On one hand there are many models coming from the real world which it can be analyzed by piecewise linear differential systems. It seems that piecewise linear differential systems can have almost all the dynamical behaviour which occur in the smooth ones, see for instance [1, 22]. On the other hand it is showed that most of the piecewise linear differential systems can be

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