PERIODIC ORBITS FOR THE GENERALIZED YANG-MILLS HAMILTONIAN SYSTEM IN DIMENSION 6: II

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Abstract. We complete the analytical study of the families of periodic orbits started in the paper [14] for the generalized Yang-Mills Hamiltonian system in dimension 6 using the averaging theory.

1. Introduction and statements of the main results

We study a generalized classical Yang–Mills Hamiltonian system in dimension 6. It is formed by a harmonic oscillator plus the more general homogeneous potential of fourth degree with monomials having only even powers.

$$H = \frac{1}{2}(p_x^2+p_y^2+p_z^2+x^2+y^2+z^2)+\frac{1}{4}(ax^4+2bx^2y^2+2cx^2z^2+dy^4+2ey^2z^2+fz^4). \tag{1}$$

Note that the Hamiltonian (1) depends on six real parameters a,b,c,d,e and f. The principal aim is to study the periodic solutions in the different energy levels $H = h$ of the Hamiltonian system associated to the Hamiltonian (1).

Until now the majority of articles related with the Hamiltonian (1) studied the planar classical Yang-Mills Hamiltonian system, i.e. the Hamiltonian (1) with $z = p_z = 0$. The periodic solutions of the planar system were studied in [15]. Contopoulos and co-workers during many years studied this planar Yang-Mills Hamiltonian system with $a = 0$. This Hamiltonian is now known as the Contopoulos Hamiltonian which describes the perturbed central part of an elliptical or barred galaxy without escapes. For more details see the references [6], [7], and [8]. Deprit and Elipe in [9] studied also several periodic orbits and bifurcations for this planar Hamiltonian system. When $d = 0$ and the quadratic part $(x^2 + y^2)/2$ is eliminated from the Hamiltonian we obtain the mechanical Yang-Mills Hamiltonian $H = (p_x^2+p_y^2)/2+bx^2y^2/2$. Many authors studied this quartic homogeneous potentials (without quadratic terms), For more details see the references [3], [4] and [11]. Moreover, if $b \neq 0$ the Hamiltonian of Yang-Mills is well known that it is non integrable and strongly chaotic. Other researches associated to this mechanical system of Yang-Mills with quartic potencials having three up to five terms were treated in [5], [10], [13], and [17]. Maciejewski et. al. [17] studied generalized Yang-Mills Hamiltonian systems having a quadratic potential plus a homogeneous potential of fourth degree with five parameters. they proved the existence of connected branches of non stationary periodic trajectories starting at the origin. Caranicolas and Varvoglis [5] studied a Hamiltonian with a quartic potential of three parameters plus a quadratic harmonic potential with two frequencies ω_1 and ω_2 of the form

$$H = \frac{1}{2}(p_x^2 + p_y^2 + \omega_1^2x^2 + \omega_2^2y^2) + \varepsilon(ax^4 + 2bx^2y^2 + cy^4).$$

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