3-DIMENSIONAL ZERO-HOPF BIFURCATION VIA AVERAGING THEORY

SARA KASSA [1], JAUME LLIBRE [2] AND AMAR MAKHLOUF[1]

Abstract. By using the averaging theory of second order we study the existence of limit cycles bifurcating from a zero-Hopf equilibrium for a general polynomial differential systems in $\mathbb{R}^3$ with cubic homogeneous nonlinearities. The result obtained shows that at least 3 limit cycles can bifurcate from a such zero-Hopf equilibrium.

1. Introduction and statement of the main result

Our goal is to study the periodic solutions which can bifurcate from a zero-Hopf equilibrium of polynomial differential systems in $\mathbb{R}^3$ with cubic nonlinearities by using the averaging theory of the second order. A *zero-Hopf equilibrium* in $\mathbb{R}^3$ is an equilibrium point having eigenvalues 0 and $\pm bi$ with $b \neq 0$.

In [7] where studied the number of periodic solutions which bifurcate from a zero-Hopf equilibrium of a polynomial differential system in $\mathbb{R}^n$ for $n \geq 2$ using the averaging theory of first order. They obtained that 2^{n-3} periodic solutions can bifurcate from an equilibrium point with eigenvalues $\pm bi$ with $b \neq 0$ and $n-2$ zeros.

In [5] the authors studied the zero-Hopf bifurcation occurring in polynomial differential systems in $\mathbb{R}^3$ with quadratic homogeneous nonlinearities by using averaging theory of second order, and they obtain that at least 3 limit cycles can bifurcate from the zero-Hopf bifurcation.

In [3] the authors studied the zero-Hopf bifurcation of the polynomial differential systems in $\mathbb{R}^4$ with quadratic nonlinearities obtaining at least 9 limit cycles bifurcating from zero-Hopf equilibrium point using the averaging theory of second order.

In this paper we are interested on the existence of periodic solutions which can bifurcate from a zero-Hopf equilibrium point of a polynomial differential system in $\mathbb{R}^3$ with cubic homogeneous nonlinearities. More precisely we consider differential systems of the form

2010 *Mathematics Subject Classification.* Primary: 34C07, 34C05, 34C40.

Key words and phrases. Hopf bifurcation, averaging theory, cubic polynomial differential systems.