ON THE CROSSING LIMIT CYCLES FOR PIECEWISE LINEAR DIFFERENTIAL SYSTEMS SEPARATED BY A STRAIGHT LINE AND HAVING SYMMETRIC EQUILIBRIUM POINTS

JOHANA JIMENEZ[1], JAUME LLIBRE[2] AND JOÃO C. MEDRADO[3]

Abstract. In this paper we study the maximum number of crossing limit cycles that can have the planar piecewise linear differential systems separated by a straight line Σ and formed by two linear differential systems X^-, X^+ which singularities are symmetrical with respect to the straight line of discontinuity Σ. More precisely, the singularities points of the linear differential systems X^-, X^+ considered can be a center (C), a focus (F), a diagonalizable node (N), an improper node (iN) or a saddle (S), which can be real or virtual. Then we have fourteen cases depending of the type and the position of the singularities of X^- and X^+. Here we provide lower or upper bounds for the maximum number of crossing limit cycles for each case.

1. Introduction and statement of the main results

The qualitative theory of discontinuous piecewise differential systems arose in a natural way in the study of nonlinear oscillations by Andronov, Vitt and Khaikin in [1]. Moreover in these last years this qualitative theory is a matter of great interest for many researchers because these systems are used to investigate nonlinear dynamics, to model several real phenomena like cell activity and processes appearing in electronics, mechanics, economy, etc., see for instance [3, 5, 21, 24] and references quoted therein.

We recall that a *crossing limit cycle* is a periodic orbit isolated in the set of all periodic orbits of the piecewise linear differential system, which only have isolated points of intersection with the discontinuity curve.

The class of piecewise linear differential systems in $\mathbb{R}^2$ with two zones separated by a straight line Σ is the simplest class of piecewise differential systems. We can consider without loss of generality that the discontinuity straight line is $\Sigma = \{(x,y) \in \mathbb{R}^2 : x = 0\}$. It separates the plane into two regions, namely

$$\Sigma^- = \{(x,y) \in \mathbb{R}^2 : x < 0\} \text{ and } \Sigma^+ = \{(x,y) \in \mathbb{R}^2 : x > 0\}.$$

Therefore we obtain the piecewise linear differential system

$$\dot{X} = \begin{cases} X^- = A^-X + B^-, & \text{if } (x,y) \in \Sigma^-, \\ X^+ = A^+X + B^+, & \text{if } (x,y) \in \Sigma^+, \end{cases} \tag{1}$$

where

$$A^\pm = \begin{pmatrix} a_{11}^\pm & a_{12}^\pm \\ a_{21}^\pm & a_{22}^\pm \end{pmatrix}, \quad B^\pm = \begin{pmatrix} b_1^\pm \\ b_2^\pm \end{pmatrix} \text{ and } X = (x,y)^T \in \mathbb{R}^2$$

In [20] Lum and Chua conjectured that a continuous piecewise linear differential system (1) has at most one crossing limit cycle. In [9] Freire et al. proved this conjecture. There are several papers tried to investigate the problem of Lum and Chua for the class of discontinuous piecewise linear differential systems in the plane. For instance in [10] Han and

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