NEW FAMILIES OF PLANAR HAMILTONIAN SYSTEMS HAVING GLOBAL CENTERS

JACKSON ITIKAWA[1] AND JAUME LLIBRE[2]

ABSTRACT. Let $F = (P, Q) : U \to \mathbb{R}^2$ be a $\mathcal{C}^2$ function, with U an open connected set of $\mathbb{R}^2$, such that the Jacobian of F never vanishes. We present new families of Hamiltonian systems with Hamiltonian $H = \left(P(x, y)^2 + Q(x, y)^2\right)/2$ having a global center in U.

Moreover if P and Q are analytic functions in U and the Jacobian of F is a non–zero constant, then the global centers of these Hamiltonian systems in U are isochronous.

1. Introduction and Statement of the Main Results

Consider a planar $\mathcal{C}^k$ differential system, i.e. a differential system of the form

$$\dot{x} = f(x, y), \qquad \dot{y} = g(x, y), \tag{1}$$

where $f, g : U \to \mathbb{R}^2$ are $\mathcal{C}^k$ functions, being U the maximum open connected set of $\mathbb{R}^2$ where f and g are defined.

A singular point p of system (1) is a *center* if there is a neighborhood $V_p \subset U$ such that all solutions in $V_p \setminus \{p\}$ are periodic. The maximal open connected set $W_p \subset U$ formed by periodic orbits surrounding the center p and having p as its inner boundary is called the *period annulus of the center*. When $W_p = U$ we have a *global center*. If all the periodic solutions of W_p have the same period, we say that the center is *isochronous*.

Let q be a singular point of the differential system (1) and $F = (P, Q)$. As usual we denote the Jacobian matrix F at the point q by $DF(q)$. Then q is a *non-degenerate singular point* if the determinant of $DF(q)$ does not vanish. In such a case a necessary condition in order that q be a center is that the eigenvalues of the matrix $DF(q)$ be pure

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