HAMILTONIAN POLYNOMIAL DIFFERENTIAL SYSTEMS WITH GLOBAL CENTERS IN $\mathbb{R}^2$

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Abstract. We characterize the polynomial Hamiltonian systems having a global center in $\mathbb{R}^2$, and show that the polynomial Hamiltonian systems of degree $n \geq 3$ having a global center can exhibit one of all kinds of center: linear type, nilpotent or degenerate.

In particular we characterize all the cubic polynomial Hamiltonian systems having a degenerate center, and provide an approach using dynamical systems for characterizing when real algebraic curves $H(x,y) = h$ in $\mathbb{R}^2$ are a continuum of ovals varying $h \in \mathbb{R}$.

1. Introduction and statement of the main results

Let $H(x,y)$ be a polynomial of degree $n+1$ in the variables x and y with coefficients in $\mathbb{R}$, then the polynomial differential system

$$(1) \qquad \dot{x} = -\frac{\partial H(x,y)}{\partial y} = P(x,y), \qquad \dot{y} = \frac{\partial H(x,y)}{\partial x} = Q(x,y),$$

is called a *polynomial Hamiltonian system of degree* n *with Hamiltonian* $H(x,y)$, where n is a positive integer, denoted by $n \in \mathbb{N}$.

The notion of center goes back to Poincaré [17] and Dulac [9]. A *center* is an equilibrium point p of system (1) in the plane $\mathbb{R}^2$, which has a neighborhood U such that p is the unique equilibrium in U and $U \setminus \{p\}$ is filled by periodic orbits (closed orbits or ovals) enclosing p. The center p is *global* if $\mathbb{R}^2 \setminus \{p\}$ is filled by periodic orbits.

To characterize the real algebraic curves $H(x,y) = h$ in $\mathbb{R}^2$ having ovals for a continuum of the values of $h \in \mathbb{R}$, is equivalent to characterize the centers of the Hamiltonian system (1) in $\mathbb{R}^2$ with the polynomial Hamiltonian function $H(x,y)$. For a center of system (1) with Hamiltonian $H(x,y)$ we can define its *period function* $T(h)$ as the period of the periodic orbit contained in the curve $H(x,y) = h$.

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