PERIODIC SOLUTIONS AND THEIR STABILITY FOR SOME PERTURBED HAMILTONIAN SYSTEMS

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ABSTRACT. We deal with non–autonomous Hamiltonian systems of one degree of freedom. For such differential systems we compute analytically some of their periodic solutions, together with their type of stability. The tool for proving these results is the averaging theory of dynamical systems. We present some applications of these results.

1. INTRODUCTION AND MAIN RESULTS

We consider the following perturbed first order differential systems

$$\frac{dp}{dt} = -\frac{\partial \mathcal{H}}{\partial q} + \varepsilon \mathcal{P}_1(q,p,t), \qquad \frac{dq}{dt} = \frac{\partial \mathcal{H}}{\partial p} + \varepsilon \mathcal{P}_2(q,p,t), \tag{1}$$

where $\mathcal{H} = \mathcal{H}(q,p)$ is a Hamiltonian function defined in an open set U of $\mathbb{R}^2$, q is the position, p its associated momentum, the functions $\mathcal{P}_i$ defined in $U \times \mathbb{R}$ are smooth $2\pi-$ periodic in t, and ε is a small parameter.

All the lemmas, theorems and corollaries stated in this section are proved in the next two sections. The theorems are proved using the averaging theory, see in the appendix a summary of the results on this theory that we need for proving our theorems. For computing analytically periodic solutions of the differential equations we shall use the averaging theory, see for instance [2, 3, 4], but in those papers we studied periodic solutions of autonomous Hamiltonian systems, and in the present one we are working with non-autonomous differential systems.

We assume that the Hamiltonian $\mathcal{H}$ is expressed in action–angle variables (I,θ) as $\mathcal{H}(p,q) = \mathcal{H}_0(I)$. If the change of variable $(p,q) \mapsto (I,\theta)$ is given by $I = I(p,q)$ and $\theta = \theta(p,q)$, then we consider the functions

$$\mathcal{F}_1(I,\theta,t) = \frac{\partial I}{\partial p}\mathcal{P}_1(q,p,t) + \frac{\partial I}{\partial q}\mathcal{P}_2(q,p,t),$$
$$\mathcal{F}_2(I,\theta,t) = \frac{\partial \theta}{\partial p}\mathcal{P}_1(q,p,t) + \frac{\partial \theta}{\partial q}\mathcal{P}_2(q,p,t).$$

The following result holds taking into account that the change of variables to action–angle variables is canonical.

1991 *Mathematics Subject Classification.* [2010]Primary 34C10, 34C25.
Key words and phrases. Hamiltonian system, periodic solutions, averaging theory.