LOCAL CYCLICITY IN LOW DEGREE PLANAR PIECEWISE POLYNOMIAL VECTOR FIELDS

LUIZ F. S. GOUVEIA AND JOAN TORREGROSA

Abstract. In this work, we are interested in isolated crossing periodic orbits in planar piecewise polynomial vector fields defined in two zones separated by a straight line. In particular, in limit cycles of small amplitude. They are all surrounding one equilibrium point or a sliding segment. We provide lower bounds for the local cyclicity for planar piecewise polynomial systems, $M_p^c(n)$, with degrees 2, 3, 4 and 5. More concretely, $M_p^c(2) \geq 13$, $M_p^c(3) \geq 26$, $M_p^c(4) \geq 40$ and $M_p^c(5) \geq 58$. Clearly, all of them are in only one nest. The computations use a parallelization algorithm.

1. Introduction

The study of piecewise or non-smooth systems was started by the school of Andronov, see for example [3]. Many problems of engineering can be modeled by this class of systems, see [1]. Recently, they also appear modeling different situations in physics and biology, see [14]. One of the most studied situations in the plane is given by two vector fields defined in two half-planes separated by a straight line. As in the case of the classical qualitative theory of polynomial systems, the study of the number and location of the isolated periodic orbits, also called limit cycles, have received special attention. See for example [6, 12, 19, 24, 27, 30]. In particular, it can be seen as an extension of the 16th-Hilbert problem for planar piecewise polynomial vector fields. Ilyashenko, in [26], presented an updated summary of the status of this problem, proposed by Hilbert more than one hundred years ago.

Our main interest, in this work, is the study of the number of limit cycles bifurcating from the origin in the class of piecewise differential equations. In particular, the ones that write as

$$\begin{cases} (x', y') = (P^+(x,y,\lambda), Q^+(x,y,\lambda)), \text{ when } y \geq 0, \\ (x', y') = (P^-(x,y,\lambda), Q^-(x,y,\lambda)), \text{ when } y < 0, \end{cases} \tag{1}$$

where $P^\pm(x,y,\lambda)$ and $Q^\pm(x,y,\lambda)$ are polynomials. The straight line $\Sigma = \{y = 0\}$ divides the plane in two half-planes $\Sigma^\pm = \{(x,y) : \pm y > 0\}$ and the trajectories on Σ are defined following the Filippov convention, see [15]. The so-called crossing limit cycles are the ones that, when they pass through the separation line Σ, both vector fields point out in the same direction.

For polynomial vector fields of degree n, as usual, we denote by $M(n)$ the maximum number of limit cycles bifurcating from a monodromic singular point and by $H(n)$ the maximum number of limit cycles. For piecewise polynomial vector fields, also of degree n, we denote respectively both numbers by $M_p^c(n)$ and $H_p^c(n)$. The upper index c means crossing limit cycles. Clearly, $M(n) \leq H(n)$ and $M_p^c(n) \leq H_p^c(n)$. Clearly, linear systems have no limit cycles, then $H(1) = M(1) = 0$. While they appear in piecewise linear systems defined in two zones. For the class of piecewise linear systems defined in

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