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PERIODS OF PERIODIC HOMEOMORPHISMS OF PINCHED SURFACES WITH ONE OR TWO BRANCHING POINTS

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Abstract. In this paper we characterize all the possible sets of periods of a periodic homeomorphism defined on compact connected pinched surfaces with one or two branching points.

1. Introduction

A *pinched surface* $\mathbb{S}$, here studied, is a compact set formed by one or two *vertices* (points) and *handles*, where here a handle is homeomorphic to an open cylinder, i.e. to the set $(0,1)\times\mathbb{S}^1$, where $(0,1)$ is the open interval of the real line and $\mathbb{S}^1$ is the circle. The boundaries of every handle are vertices. Furthermore, the handles are pairwise disjoint, and the pinched surfaces that we consider here will always be connected.

Let $\mathbb{S}$ be a pinched surface and let $z\in\mathbb{S}$ be a vertex. We consider a small open neighborhood U (in $\mathbb{S}$) of z. The number of connected components of $U\setminus\{z\}$ is called the *valence* of z and is denoted by $\mathrm{Val}(z)$. Observe that this definition is independent of the choice of U if U is sufficiently small. A vertex of valence 1 is called an *endpoint of* $\mathbb{S}$ and a vertex of valence larger than 1 is called a *branching point of* $\mathbb{S}$.

A continuous map $f:\mathbb{S}\to\mathbb{S}$ is called *periodic* if there exists a positive integer n such that the iterate f^n is the identity map, i.e. $f^n(x)=x$ for all $x\in S$, or $f=id$.

Let $f:\mathbb{S}\to\mathbb{S}$ be a continuous map. A point $z\in S$ such that $f(z)=z$ is called a *fixed point*, or a periodic point of period 1. The point $z\in\mathbb{S}$ is *periodic* of *period*

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