DYNAMICS AND DARBOUX INTEGRABILITY OF THE D_2 POLYNOMIAL VECTOR FIELDS OF DEGREE 2 IN $\mathbb{R}^3$

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ABSTRACT. We characterize the Darboux integrability and the global dynamics of the 3-dimensional polynomial differential systems of degree 2 which are invariant under the D_2 symmetric group, which have been partially studied previously by several authors.

1. Introduction and statement of the results

Differential systems having some symmetries appear often in many applications, and consequently have been studied by several authors, see for instance [6, 8, 9, 12].

In this paper we shall study the dynamics of the 3-dimensional autonomous polynomial differential systems of degree 2 symmetric with respect to the group of symmetries generated by the following transformations of $\mathbb{R}^3$

$$(x,y,z)\mapsto(x,y,z),\quad (x,y,z)\mapsto(-x,-y,z),$$
$$(x,y,z)\mapsto(-x,y,-z),\quad (x,y,z)\mapsto(x,-y,-z).$$

In [11] it is proved that such 3-dimensional autonomous systems are

$$\dot{x}=ax+yz,\quad \dot{y}=by+xz,\quad \dot{z}=z+xy, \tag{1}$$

and

$$\dot{x}=ax+yz,\quad \dot{y}=by+xz,\quad \dot{z}=z-xy, \tag{2}$$

where a,b are non-zero parameters and the dot means derivative in t. According to [2] systems (1) and (2) are equivariant under the D_2 symmetry group.

Note that system (2) can be transformed into system (1) by the change

$$(x,y,z)\to(iX,iY,Z) \tag{3}$$

with $X,Y,Z\in\mathbb{R}$. Hence it is enough to study the integrability of system (1). Moreover, system (1) is invariant by the change

$$(a,b,x,y,z)\to(b,a,y,x,z). \tag{4}$$

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