A NEW SUFFICIENT CONDITION IN ORDER THAT THE REAL JACOBIAN CONJECTURE IN $\mathbb{R}^2$ HOLDS

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Abstract. Let $F = (f, g) : \mathbb{R}^2 \to \mathbb{R}^2$ be a polynomial map such that $\det(DF(x, y))$ is nowhere zero and $F(0, 0) = (0, 0)$. In this work we give a new sufficient condition for the injectivity of F. We also state a conjecture when $\det(DF(x, y)) = \text{constant} \neq 0$ and $F(0, 0) = (0, 0)$ equivalent to the Jacobian conjecture.

1. Introduction and statement of the main result

Let $F = (f, g) : \mathbb{R}^2 \to \mathbb{R}^2$ be a smooth map such that the determinant of the Jacobian matrix $\det(DF)$ is nowhere zero. By the Inverse Theorem such a map F is a local diffeomorphism. However this map is not always an injective map. But with some additional conditions it holds that F is a global diffeomorfphism, see for instance [9, 12, 20].

The *real Jacobian conjecture*, stated by Keller [18] in 1939, says that when F is a polynomial map, then F is injective. However in 1994 Pinchuk [19] gaves a counterexample to this conjecture providing a non injective polynomial map with nonvanishing Jacobian determinant. Nevertheless with additional conditions the conjecture holds, for instance in [3, 5] it was shown that the conjecture is true if the degree of f is at most 4. In [4] the following result provides two independent sufficient conditions for the validity of the real Jacobian conjecture.

Theorem 1. *Let $F = (f, g) : \mathbb{R}^2 \to \mathbb{R}^2$ be a polynomial map such that $\det(DF)$ is nowhere zero and $F(0, 0) = (0, 0)$. If the higher homogeneous terms of the polynomials $ff_x + gg_x$ and $ff_y + gg_y$ do not have real linear factors in common, then F is injective.*

Theorem 1 improves a preliminary result in [6] which said: if $\deg f = \deg g$ and that the homogeneous terms of higher degree of f and g do not have real linear factors in common, then F is injective. A similar

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