ON THE TAKENS-ARGÉMI-BENOÎT'S TRANSFORMATION FOR THE EXISTENCE OF CANARDS SOLUTIONS

JEAN-MARC GINOUX[1] AND JAUME LLIBRE[2]

Abstract. We show that the Jacobian matrix of the "normalized slow dynamics" can be directly deduced from the three-dimensional singularly perturbed system to which it is associated while using a transformation we call "Takens-Argémi-Benoît's transformation" which is thus expressed as a linear function of the Jacobian matrix of the original singularly perturbed system and of the Hessian of its slow invariant manifold. Then, we prove, while using well-known results concerning the determinant of the sum, that the determinant of the Jacobian matrix of the projection of the "normalized slow dynamics" on the tangent bundle to the slow manifold leads to the same condition for existence of canards solutions as that provided by Benoît in the beginning of the eighties.

1. Introduction

We consider the differential system in $\mathbb{R}^3$:

$$
\begin{aligned}
\frac{dx}{dt} &= f\left(x, y, z\right), \\
\frac{dy}{dt} &= g\left(x, y, z\right), \\
\frac{dy}{dt} &= \frac{1}{\varepsilon} h\left(x, y, z\right).
\end{aligned} \tag{1}
$$

where f, g, h are C^2 functions and ε a small parameter.

The differential system (1) can be also written as $\dot{\vec{X}} = \vec{F}(\vec{X})$ where $\vec{X} = [x, y, z]^t$ et $\vec{F}(\vec{X}) = \left[f\left(x, y, z\right), g\left(x, y, z\right), \varepsilon^{-1} h\left(x, y, z\right)\right]^t$.

By using Non-Standard Analysis, Callot *et al.* [8], Benoît *et al.* [2], and then Benoît and Lobry [3] and finally Benoît [4, 7] have proved the generic existence of solutions of the singularly perturbed system (1) in $\mathbb{R}^3$ that they called "canards solutions". A few years later, Szmolyan and Wechselberger [14] and then Wechselberger [16, 17] have stated the same result while using the so-called *Geometric Singular Perturbation Theory* providing thus a "standard" version of Benoît's theorem [4, 7]. Both methods aims to project on the one hand the "normalized slow dynamics" on the tangent bundle of the *slow manifold* of Fenichel [10, 13] defined by $h(x, y, z) = 0$, and to prove on the other hand, that the "pseudo-singular point" is of *saddle* type (See Benoît [4, p. 167] and Szmolyan and Wechselberger [14, p.

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