LOWER BOUNDS FOR THE LOCAL CYCLICITY FOR FAMILIES OF CENTERS

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ABSTRACT. In this paper we are interested on how the local cyclicity of a family of centers depends on the parameters. This fact, was pointed out in [21], to prove that there exists a family of cubic centers, labeled by CD_{31}^{12} in [25], with more local cyclicity than expected. In this family there is a special center such that at least twelve limit cycles of small amplitude bifurcate from the origin when we perturb it in the cubic polynomial general class. The original proof has some important gaps that we correct here. We take the advantage of better understanding of the bifurcation phenomenon in non generic cases to show two new cubic systems exhibiting 11 limit cycles and another exhibiting 12. Finally, using the same techniques, we study the local cyclicity of holomorfic quartic centers, proving that 21 limit cycles of small amplitude bifurcate from the origin, when we perturb in the class of quartic polynomial vector fields.

1. INTRODUCTION

The study of limit cycles began at the end of the 19th century with Poincaré. Years later, in 1900, Hilbert presents a list of unsolved problems. From the original 23 problems of this list, the 16th is still open. The second part of this problem consists in determining a uniform bound of the maximal number of limit cycles (named $H(n)$), and their relative positions, of a planar polynomial systems of degree n. However, there are also weak versions of 16th Hilbert's problem. Arnold in [1] proposed a version focused on studying the number of limit cycles bifurcating from the period annulus of Hamiltonians systems. In this paper, we are interested in provide the maximal number $M(n)$ of small amplitude limit cycles bifurcating from an elementary center or an elementary focus, in special for degrees 3 and 4. The main idea is to study the local cyclicity of families of centers depending on a finite number of parameters.

As it is well known, for $n = 2$, Bautin proved in [2] that $M(2) = 3$. The case $n = 3$ but without quadratic terms (homogeneous cubic perturbation) was studied in [3, 19] and solved in [23], then $M_h(3) = 5$. In [24, 26] Zoladek shown that $M(3) \geq 11$. Christopher, in [5], gave a simple proof of Zoladek's result perturbing another cubic center with a rational first integral, using only the linear parts of the Lyapunov constants. The interest of this result is that we can compute these linear parts [5], in a parallelized way [14, 17], near a center without having the complete expressions of the Lyapunov constants. Basically the used technique consists in to choose a point on the center variety and at this point consider the linear term of the Lyapunov constants, if the point is chosen on a component of the center variety of codimension k, then the first k linear terms of the Lyapunov constants are

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