Limit cycles of continuous piecewise differential systems formed by linear and quadratic isochronous centers I

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First we study the planar continuous piecewise differential systems separated by the straight line $x = 0$ formed by a linear isochronous center in $x > 0$ and an isochronous quadratic center in $x < 0$. We prove that these piecewise differential systems cannot have crossing periodic orbits, and consequently they do not have crossing limit cycles.
Second we study the crossing periodic orbits and limit cycles of the planar continuous piecewise differential systems separated by the straight line $x = 0$ having in $x > 0$ the general quadratic isochronous center $\dot{x} = -y + x^2 - y^2$, $\dot{y} = x(1 + 2y)$ after an affine transformation, and in $x < 0$ an arbitrary quadratic isochronous center. For these kind of continuous piecewise differential systems the maximum number of crossing limit cycles is one, and there are examples having one crossing limit cycles
In short for these families of continuous piecewise differential systems we have solved the extension of the 16th Hilbert problem to themselves.

Keywords: Limit cycles, isochronous quadratic centers, continuous piecewise linear differential systems, first integrals

1. Introduction

In the qualitative theory of planar differential systems a *limit cycle* is an isolated periodic solution in the set of all periodic solutions, which remained the most sought solutions when modeling physical systems in the plane. As far as we known the notion of limit cycle appeared in the year 1885 in the work of Poincaré [Poincaré, 1928].

Most of the early examples in the theory of limit cycles in planar differential systems were commonly related to practical problems with mechanical and electronic systems, but periodic behavior appears in all branches of the sciences. To determine the existence of non–existence of limit cycles is one of the more difficult objects in the qualitative theory of planar differential equations. A large amount of references deals with the subject of limit cycles, many of them motivated for the famous second part of the Hilbert's 16th problem, which ask for the maximum number of limit cycles that the planar polynomial differential systems of a given degree can exhibit, see for details [Hilbert, 1900; Ilyashenko, 2002; Li, 2003].

Since 1930's the study of the limit cycles also became important in the continuous and discontinuous piecewise differential systems separated by a straight line, due to their applications to mechanics, electrical