



# A Chebyshev criterion with applications

A. Gasull<sup>a</sup>, A. Geyer<sup>b,\*</sup>, F. Mañosas<sup>a</sup>

<sup>a</sup> *Departament de Matemàtiques, Universitat Autònoma de Barcelona, 08193 Bellaterra, Barcelona, Spain*

<sup>b</sup> *Delft Institute of Applied Mathematics, Faculty of Electrical Engineering, Mathematics and Computer Science, Delft University of Technology, Van Mourik Broekmanweg 6, 2628 XE Delft, the Netherlands*

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## Abstract

We show that a family of certain definite integrals forms a Chebyshev system if two families of associated functions appearing in their integrands are Chebyshev systems as well. We apply this criterion to several examples which appear in the context of perturbations of periodic non-autonomous ODEs to determine bounds on the number of isolated periodic solutions, as well as to persistence problems of periodic solutions for perturbed Hamiltonian systems.

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## 1. Main results

Chebyshev systems ( $T$ -systems), complete Chebyshev systems ( $CT$ -systems) and extended complete Chebyshev systems ( $ECT$ -systems) are the natural extensions of polynomials of a given degree  $m$  to more general functions. Notice that degree  $m$  polynomials can be seen as elements of the vector space  $\langle 1, x, \dots, x^m \rangle$  of dimension  $m + 1$ , for which each element has at most  $m$  roots, counting multiplicities, such that this bound is attained. In the next section we give

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\* Corresponding author.

E-mail address: [a.geyer@tudelft.nl](mailto:a.geyer@tudelft.nl) (A. Geyer).