A CHEBYSHEV CRITERION WITH APPLICATIONS

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ABSTRACT. We show that a family of certain definite integrals forms a Chebyshev system if two families of associated functions appearing in their integrands are Chebyshev systems as well. We apply this criterion to several examples which appear in the context of perturbations of periodic non-autonomous ODEs to determine bounds on the number of isolated periodic solutions, as well as to persistence problems of periodic solutions for perturbed Hamiltonian systems.

1. MAIN RESULTS

Chebyshev systems (T-systems), complete Chebyshev systems (CT-systems) and extended complete Chebyshev systems (ECT-systems) are the natural extensions of polynomials of a given degree m to more general functions. Notice that degree m polynomials can be seen as elements of the vector space $\langle 1, x, \dots, x^m \rangle$ of dimension $m+1$, for which each element has at most m roots, counting multiplicities, such that this bound is attained. In the next section we give the precise definition of T, CT and ECT-systems, which essentially introduce them as vector spaces of functions satisfying these properties.

When studying perturbations of Hamiltonian systems with a continuum of periodic orbits, the level sets of the periodic orbits that persist as limit cycles are given by the zeroes of a line integral, or *Abelian integral*, see for instance [8]. A commonly used method to control the number of zeroes of such integrals when the perturbation depends on parameters is to prove that they form a basis which is a Chebyshev system. With this aim, a criterion was developed in [6] which shows that if some functions constructed from the integrands of the Abelian integrals form a Chebyshev system, then the Abelian integrals that generate the complete Abelian integral itself are a Chebyshev system as well. This is a powerful result since proving the Chebyshev property for functions is usually easier than to do so for functions defined as line integrals.

In the same spirit, the goal of the present paper is to prove that some definite integrals form a Chebyshev system if families of functions given by the integrands of these integrals are also Chebyshev systems. Our main result is: