SOME OPEN PROBLEMS IN LOW DIMENSIONAL DYNAMICAL SYSTEMS

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Abstract. The aim of this paper is to share with the mathematical community a list of 33 problems that I have found along the years during my research. I believe that it is worth to think about them and, hopefully, it will be possible either to solve some of the problems or to make some substantial progress. Many of them are about planar differential equations but there are also questions about other mathematical aspects: Abel differential equations, difference equations, global asymptotic stability, geometrical questions, problems involving polynomials or some recreational problems with a dynamical component.

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1. Introduction

There are several famous well-known conjectures and open problems, like for instance Jacobian conjecture, Riemman's conjecture, $3x+1$ conjecture or Collatz problem, Goldbach's conjecture, or Hilbert XVI problem, that almost all mathematicians know. Also a very interesting list of 18 open problems, covering many different branches of mathematics, has been published by Smale, see [122]. The aim of this work is much more modest. I will list several concrete problems that I have found along the years. I hope that, at least for some of them, it is possible either to solve or to make some substantial progress.

The problems will be classified in seven categories: periodic orbits, period function, piecewise linear systems, Markus-Yamabe and La Salle problems, geometrical problems, questions involving polynomials, and recreational questions with a dynamical flavour. Next we briefly describe them but without precise definitions. In the corresponding next sections they are contextualized and stated with more precision.

In Section 2 we will propose some questions about the maximum number of limit cycles of some low dimensional differential equations, including rigid systems, homogeneous type differential systems, Liénard systems, Riccati and Abel differential equations, and a new point of view of Hilbert's XVI problem. Some other related questions considered in this section are on a second order singular differential equation, about the maximum number of centers for polynomial differential systems and on the characterization of some rational periodic difference equations.

In Section 3 we propose several problems for the period function of some families of planar systems: a Hamiltonian one, a system with homogenous components, a third one about the maximum number of critical periods for planar polynomial differential systems, and we end with the problem proposed by Chicone about the maximum number of critical periods for quadratic reversible centers and with a related one about the period function of a family of reversible equivariant planar differential systems.