# ANALYTIC INTEGRABILITY OF QUASI-HOMOGENEOUS SYSTEMS VIA THE YOSHIDA METHOD

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Abstract. The objective of this paper is double. First we do a survey on what we call the Yoshida method for studying the analytic first integrals of the quasi-homogeneous polynomial differential systems. After we apply the Yoshida method for studying the analytic first of all the quasi-homogeneous polynomial differential systems in $\mathbb{R}^3$ of degree 2.

## 1. Introduction

In 1983 Haruo Yoshida [15, 16] publishes a series of interesting results that establish conditions for the integrability of some classes of differential systems and provide a way for finding first integrals for such systems. Later on several authors [1, 3, 8, 12, 9, 11, 14] have continued to develop his ideas until to have what we call now the *Yoshida method*.

In essence the method is based on the correspondence between certain characteristic values of the first integrals and others inherent to the differential system (the so-called *Kowalevskaya exponents*), being all of them calculable in a finite number of steps.

The main purpose of this work is to analyze the capabilities of the Yoshida method as a tool for the integration of quasi-homogeneous differential systems in the space $\mathbb{R}^3$, a class of differential systems on which these results have been little exploited so far. Additional to this analysis of the Yoshida method, another of our objectives is to carry out a compilation of the main results on this class of quasi-homogeneous differential systems published to date on the subject.

Consider an $n$-dimensional autonomous polynomial differential system of the form

$$\frac{dx_i}{dt} = \dot{x_i} = P_i(\mathbf{x}), \qquad \mathbf{x} = (x_1, ..., x_n) \in \mathbb{R}^n, \qquad i = 1, ..., n, \tag{1}$$

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