VANISHING SET OF INVERSE JACOBI MULTIPLIERS AND ATTRACTOR/REPULSOR SETS

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Abstract. In this paper we study conditions under which the zero-set of the inverse Jacobi multiplier of a smooth vector field contains its attractor/repulsor compact sets. The work generalizes previous results focusing on sink singularities, orbitally asymptotic limit cycles and monodromic attractor graphics. Taking different flows on the torus and the sphere as canonical examples of attractor/repulsor sets with different topologies, several examples are constructed illustrating the results presented.

1. Introduction and main results

We consider C^1 autonomous ordinary differential equations $\dot{x} = f(x)$ defined in $\mathbb{R}^n$ where $x = (x_1, \dots, x_n) \in \mathbb{R}^n$ and its associated vector field $\mathcal{X} = \sum_{i=1}^n f_i(x)\partial_{x_i}$ where $f(x) = (f_1(x), \dots, f_n(x))$. We denote by $\text{sing}(\mathcal{X})$ the set of singular points of $\mathcal{X}$ and by $\text{div}\mathcal{X} = \sum_{i=1}^n \partial f_i(x)/\partial x_i$ its divergence.

Let ϕ_t be the flow associated to $\mathcal{X}$ with ϕ_0 the identity function. A set $\Gamma \subset \mathbb{R}^n$ is *invariant* if $\phi_t(\Gamma) = \Gamma$ for any t for which the flow is defined. We define the distance from a point $p \in \mathbb{R}^n$ to the set Γ as $d(p;\Gamma) = \inf\{d(p,q) : q \in \Gamma\}$ where $d(p,q)$ is the Euclidean distance between the points p and q. If $d(\phi_t(p);\Gamma) \to 0$ when $t \to \infty$ we will write $\phi_t(p) \to \Gamma$ as $t \to \infty$. The *basin of attraction* of a set Γ is the set $\{p \in \mathbb{R}^n : \phi_t(p) \to \Gamma\}$, that is the set of initial conditions that approaches to Γ asymptotically under the forward flow. The basin of attraction is an open set.

There are several equivalent ways to define an attractor set. In [12] and [13] Milnor introduced a definition of attractor. A set $\Gamma \subset \mathbb{R}^n$ is an *attractor set* if it possesses a compact neighborhood $U \subset \mathbb{R}^n$ and there is a positive increasing time sequence $\{t_n\}_{n\in\mathbb{N}} \subset \mathbb{R}^+$ with $t_n \to \infty$ such that $\Gamma = \cap_{n\in\mathbb{N}}\phi_{t_n}(U)$ where the nested sequence of sets $U \supsetneq \phi_{t_1}(U) \supsetneq \phi_{t_2}(U) \supsetneq \cdots$ holds, this U is called a *trapping neighborhood* of Γ. This intersection Γ is always invariant, $\phi_t(\Gamma) = \Gamma$.

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