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Global dynamics of Hořava–Lifshitz cosmology with non-zero curvature and a wide range of potentials

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Abstract The global dynamics of a cosmological model based on Hořava–Lifshitz gravity in the presence of curvature is described by using the qualitative theory of differential equations.

1 Introduction

In recent years Hořava [1] proposed a spacetime asymmetric gravitational theory similar to Lifshitz's scalar field theory, also known as Hořava–Lifshitz gravity. This theory has inspired a great deal of research for its applications in cosmology and black hole physics (see [2–18] or the review articles [19,20] and the references therein).

With or without detailed-balance condition, few researchers have published several papers on Hořava–Lifshitz gravity through phase space analysis, see for instance [3–7,17]. Among these papers there are also some of them about the case of non-zero space curvature k. When the terms related to k are dominant, Friedmann solution will be generated. On the contrary, when the effective Hořava radiation dominates, the corresponding cosmology gives a solution of "radiation-dominated like" [6]. According to whether the cosmological constant Λ is zero in flat ($k = 0$) and non-flat ($k \neq 0$) universe, Leon et al. [3–5,14] divided the Hořava–Lifshitz gravity into four cases: (1) $k = 0, \Lambda = 0$; (2) $k = 0, \Lambda \neq 0$; (3) $k \neq 0, \Lambda = 0$; (4) $k \neq 0, \Lambda \neq 0$, and analyzed these phase spaces. More precisely, they either studied the global dynamics of the planar case of Hořava–Lifshitz gravity with exponential potential by using the two-dimensional Poincaré compactification, or discussed only local dynamics of this cosmological model with power-law potential but without investigate the dynamics close to infinity. The three-dimensional global dynamics of Hořava–Lifshitz cosmology with $k = 0$ and $\Lambda = 0$ was studied in [7].

In this paper we will investigate the global dynamics of the Hořava–Lifshitz scalar field cosmology under the Friedmann–Lemaître–Robertson–Walker background spacetime in the presence of curvature and no cosmological constant term, i.e., $k \neq 0$ and $\Lambda = 0$.

2 The cosmological equations

To describe the cosmological equations in this section we first give a brief review of the Hořava–Lifshitz gravitational theory as was proposed in [1]. In one common version of this theory [5,21], its field content can be given by a spatial scalar N, and a spatial vector N_i. They are the 'lapse' and 'shift' variables usually found in general relativity. Then the full metric can be written as

$$ds^2 = -N^2 dt^2 + g_{ij}(dx^i + N^i dt)(dx^j + N^j dt),$$
$$N_i = g_{ij} N^j, \tag{1}$$

where g_{ij} $(i, j = 1, 2, 3)$ is the spatial metric. The scaling transformation of the coordinates is in the form of $t \to l^3 t,\ x^i \to l x^i$, under which both N and g_{ij} are invariant, but N^i is scaled to $N^i \to l^{-2} N_i$.

Based on the detailed-balance condition [1], the full gravitational action of Hořava–Lifshitz is represented by

$$S_g = \int dt d^3x \sqrt{g} N \left\{ \frac{2}{\kappa^2}\left(K_{ij}K^{ij} - \lambda K^2\right) - \frac{\kappa^2}{2w^4} C_{ij}C^{ij} + \frac{\mu\kappa^2}{2w^2}\frac{\epsilon^{ijm}}{\sqrt{g}} R_{il}\nabla_j R^l_k - \frac{\mu^2\kappa^2}{8} R_{ij}R^{ij} - \frac{\mu^2\kappa^2}{8(3\lambda - 1)}\left(\frac{1-4\lambda}{4}R^2 + \Lambda R - 3\Lambda^2\right)\right\}, \tag{2}$$

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