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ON THE POLYNOMIAL SOLUTIONS OF THE POLYNOMIAL DIFFERENTIAL EQUATIONS $y\,y' = a_0(x) + a_1(x)\,y + a_2(x)\,y^2 + \ldots + a_n(x)\,y^n$

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In this paper we deal with differential equations of the form $yy' = P(x, y)$ where $y' = dy/dx$ and $P(x, y)$ is a polynomial in the variables x and y of degree n in the variable y. We provide an upper bound for the number of polynomial solutions of this class of differential equations, and for some particular classes we study properties of their polynomial solutions.

Key words : Polynomial differential equation; polynomial solution; linear differential equation; Riccati differential equation; Abel differential equation.

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1. Introduction and Statement of the Main Results

The study of given solutions (as polynomial or rational solutions) of differential equations is of main interest for understanding the set of solutions of a differential equation. Rainville [15] in 1936 characterized all the Riccati differential equations of the form $y' = a_0(x) + a_1(x)y + y^2$, where a_0 and a_1 are polynomials in x, having polynomial solutions. He also gave an algebraic method for studying these polynomial solutions.

In 1954 Campbell and Golomb [7] gave an algorithm for computing all the polynomial solutions of the differential equation $a(x)y' = a_0(x) + a_1(x)y + a_2(x)y^2$, where a, a_0, a_1, a_2 are polynomials in x. In 2006 Behloul and Cheng [3] provided another algorithm for finding all the rational solutions of the equation $a(x)y' = \sum_{i=0}^{n} a_i(x)y^i$, where a, a_i are polynomials in x.