4-DIMENSIONAL ZERO-HOPF BIFURCATION FOR POLYNOMIAL DIFFERENTIALS SYSTEMS WITH CUBIC HOMOGENEOUS NONLINEARITIES VIA AVERAGING THEORY

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Abstract. The averaging theory of second order shows that for polynomial differential systems in $\mathbb{R}^4$ with cubic homogeneous nonlinearities at least nine limit cycles can be born in a zero-Hopf bifurcation.

1. Introduction and statement of the main result

Our goal is to study the periodic solutions which can bifurcate at a zero-Hopf bifurcation in a polynomial differential systems in $\mathbb{R}^4$ with cubic homogeneous nonlinearities by using the averaging theory of the second order.

In [7] the authors studied the zero-Hopf bifurcation in dimension $n > 2$, by using the first order averaging method. They proved that at least 2^{n-3} limit cycles can bifurcate from one singularity with eigenvalues $\pm bi$ and $n-2$ zeros.

In [5] (resp. [2]) the authors studied the zero-Hopf bifurcation in polynomial differential systems in $\mathbb{R}^3$ (resp. $\mathbb{R}^4$) with quadratic homogeneous nonlinearities. By applying the averaging theory of the second order to these systems, they show that at most 3 limit cycles can bifurcate from a singular point having eigenvalues of the form $\pm bi$ and one zero (resp. two zeros). The zero-Hopf bifurcation in polynomial differential systems in $\mathbb{R}^3$ with cubic homogeneous nonlinearities has been studied recently in [3].

In this paper we are interested on the existence of periodic solutions bifurcating from the origin of coordinates of a polynomial differential systems in $\mathbb{R}^4$ with cubic homogeneous nonlinearities having eigenvalues $\pm bi$ and two zeros, i.e for the differential systems

$$(1)\qquad \begin{aligned} \dot{x} &= (a_1\varepsilon + a_2\varepsilon^2)x - (b + b_1\varepsilon + b_2\varepsilon^2)y + \sum_{j=0}^{2} \varepsilon^j X_j(x,y,z,w), \\ \dot{y} &= (b + b_1\varepsilon + b_2\varepsilon^2)x + (a_1\varepsilon + a_2\varepsilon^2)y + \sum_{j=0}^{2} \varepsilon^j Y_j(x,y,z,w), \end{aligned}$$

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