LOWER BOUNDS FOR THE NUMBER OF LIMIT CYCLES IN A GENERALIZED RAYLEIGH-LIÉNARD OSCILLATOR

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Abstract. In this paper a generalized Rayleigh-Liénard oscillator is consider and lower bounds for the number of limit cycles bifurcating from weak focus equilibria and saddle connections are provided. By assuming some open conditions on the parameters of the considered system the existence of up to twelve limit cycles is provided. More specifically, the approach consists in perform suitable changes in the sign of some specific parameters and apply Poincaré-Bendixson Theorem for assure the existence of limit cycles. In particular, the method for obtaining the limit cycles through the referred approach is explicitly exhibited. The main techniques applied in this study are the Lyapunov constants and the Melnikov method.

1. Introduction

1.1. **Historical facts and equations of Rayleigh and Liénard.** Ordinary differential equations (ODEs) have been largely studied in mathematics since the invention of Calculus back in 17th century. Since that the theory have proved to be very accurate to model real problems from mechanics movements and chemical reactions to social and financial sciences. The interest by ODEs gained even more traction after the remarkable work of Poincaré entitled *Mémoire sur les courbes définies par une équation différentielle*, see [12]. This paper, dated 1882, is consider one of the starting points of the so called qualitative theory of ODEs. In particular, Poincaré formally introduced the concept of limit cycle, an isolated periodic orbit inside the set of all periodic orbits of an ODE, and exhibited an *ad hoc* example an of ODE presenting a limit cycle without any connection to some concrete problem. However, the first reported case of a limit cycle surging from a real model ODE was probably provided by Rayleigh in 1877 due to a study on oscillations of a violin string, see [13]. Posteriorly in 1908 another example of limit cycle emerged from a series of works of Poincaré addressing wireless telegraphy, although the most recognized example of a limit cycle is due to Van der Pol on electrical circuits in 1927, see [15].

The goal of this paper is to study the existence and the number of limit cycles in a system which is a generalization of Rayleigh and Van der Pol systems. The equation proposed by Rayleigh which is nowadays known as *Rayleigh equation* is

$$\ddot{x} + ax + \varepsilon(c_3 + c_4\dot{x}^2)\dot{x} = 0 \qquad (1)$$

where ε is a small parameter. We also point to [10] for some historical facts about Rayleigh work.

For the sake of applications of non-linear systems it is usually interesting to assume that the unperturbed part of (1) has a potential of the form $V(x) = ax^2 + bx^4$ so the total energy

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