THE EXTENDED 16-TH HILBERT PROBLEM FOR DISCONTINUOUS PIECEWISE LINEAR CENTERS SEPARATED BY A NON-REGULAR LINE

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Abstract. We study the discontinuous piecewise differential systems formed by two linear centers separated by a non-regular straight line. We provide upper bounds for the maximum number of limit cycles that these discontinuous piecewise differential systems can exhibit and we show that these upper bounds are reached.

1. Introduction and statement of the main result

One of the main interesting objects in the study of differential systems are limit cycles. A *limit cycle* is a periodic orbit of the differential system isolated in the set of all periodic orbits of the system.

Limit cycles have played and are playing an important role for explaining phsyical phenomena, see for instance the limit cycle of van der Pol equation [18, 19], or the one of the Belousov-Zhavotinskii model [2, 21], etc.

The *extended* 16*th Hilbert problem*, that is, to find an upper bound for the maximum number of limit cycles that a given class of differential systems can exhibit, is in general an unsolved problem. Only for very few classes of differential system this problem has been solved. For the class of discontinuous piecewise differential systems here studied, we can obtain the solution by using the first integrals of the two linear centers which form the discontinuous piecewise differential system separated by a non-regular line.

The study of the piecewise linear differential systems goes back to Andronov, Vitt and Khaikin [1], and nowadays such systems still continue to receive the attention of many researchers. These differential systems are widely used to model processes appearing in electronics, mechanics, economy, etc., see for instance the books of di Bernardo et al. [3] and Simpson [20], the survey of Makarenkov and Lamb [17], as well as hundreds of references quoted in these last three works.

The simplest class of discontinuous piecewise differential systems are the planar ones formed by two pieces separated by a straight line having a linear differential system in each piece. Several authors have tried to determine the maximum number of limit cycles for this class of discontinuous piecewise differential systems. Thus, in one of the first papers dedicated to this problem, Giannakopoulos and Pliete [8]

2010 *Mathematics Subject Classification.* Primary 34A05. Secondary 34C05, 37C10.

Key words and phrases. discontinuous piecewise linear systems, linear centers, non-regular line.