4-DIMENSIONAL HOPF BIFURCATION VIA AVERAGING THEORY

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ABSTRACT. The averaging theory of second order shows that for a quadratic polynomial vector fields in $\mathbb{R}^4$ at least the maximum number of limit cycles which can born in a Hopf bifurcation is 9.

1. INTRODUCTION AND STATEMENT OF THE MAIN RESULT

The goal of this paper is to study the Hopf bifurcation occurring in quadratic polynomial vector fields in $\mathbb{R}^4$ using the averaging theory of second order. In [9] the authors studied the Hopf bifurcation in dimension $n > 2$, by using the first order averaging method. They proved that at least 2^{n-3} limit cycles can bifurcate from one singularity with eigenvalues $\pm bi$ and $n-2$ zeros. They proved for the first time that the number of bifurcated limit cycles in a Hopf bifurcation can grow exponentially with the dimension of the system.

We investigate the Hopf bifurcation at the origin of coordinates of the quadratic polynomial differential systems in $\mathbb{R}^4$

$$
\begin{aligned}
\dot{x} &= (a_1\varepsilon + a_2\varepsilon^2)x - (b + b_1\varepsilon + b_2\varepsilon^2)y + \sum_{j=0}^{2} \varepsilon^j X_j(x,y,z,w),\\
\dot{y} &= (b + b_1\varepsilon + b_2\varepsilon^2)x + (a_1\varepsilon + a_2\varepsilon^2)y + \sum_{j=0}^{2} \varepsilon^j Y_j(x,y,z,w),\\
\dot{z} &= (c_1\varepsilon + c_2\varepsilon^2)z + \sum_{j=0}^{2} \varepsilon^j Z_j(x,y,z,w),\\
\dot{w} &= (d_1\varepsilon + d_2\varepsilon^2)w + \sum_{j=0}^{2} \varepsilon^j W_j(x,y,z,w),
\end{aligned}
\tag{1}
$$

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