MINIMAL SETS OF LEFSCHETZ PERIODS OF THE MORSE–SMALE DIFFEOMORPHISMS ON $\mathbb{S}^n$ AND $\mathbb{S}^m \times \mathbb{S}^n$

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ABSTRACT. We study the set of periods of the Morse–Smale diffeomorphisms on the n-dimensional sphere $\mathbb{S}^n$, and on the products of two spheres of arbitrary dimension $\mathbb{S}^m \times \mathbb{S}^n$. We classify the minimal sets of Lefschetz periods for such Morse–Smale diffeomorphisms. This characterization is done using the induced maps on the homology and the parity of the dimension of the manifolds. The main tool used is the Lefschetz zeta function.

1. INTRODUCTION AND STATEMENT OF THE MAIN RESULTS

In this paper we deal with discrete dynamical systems defined by a diffeomorphism on a compact manifold.

Let M be a compact manifold, and let $f : M \to M$ be a continuous map, and denote by f^m the $m-$th iterate of f. A point $x \in M$ such that $f(x) = x$ is called a *fixed point*, or a *periodic point of period* 1 of f. A point $x \in M$ is called *periodic of period* $k > 1$ if $f^k(x) = x$ and $f^m(x) \neq x$ for all $m = 1, \ldots, k-1$, and the set formed by the iterates of x, i.e. $\{x,\ f(x),\ \ldots,\ f^{k-1}(x)\}$, is called the *periodic orbit* of the periodic point x.

As usual $\mathbb{N}$ denotes the set of all positive integers. Then $\mathrm{Per}(f)$ is the set $\{k \in \mathbb{N} : f$ has a periodic orbit of period $k\}$.

A fixed point x of a C^1 map f is called *hyperbolic* if if all the eigenvalues of $Df(x)$ have modulus different than one.

A periodic point x of f of period k is called a *hyperbolic periodic point* if it is a hyperbolic fixed point of f^k.

We denote by $\mathrm{Diff}(M)$ the space of all C^1 diffeomorphisms on a compact Riemannian manifold M. As it is well known the set $\mathrm{Diff}(M)$

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