

Differentiable invariant manifolds of nilpotent parabolic points

Clara Cufí-Cabré

Departament de Matemàtiques
Universitat Autònoma de Barcelona (UAB)
Barcelona Graduate School of Mathematics (BGSMath)
08193 Bellaterra, Barcelona, Spain
clara@mat.uab.cat

Ernest Fontich

Departament de Matemàtiques i Informàtica
Universitat de Barcelona (UB)
Barcelona Graduate School of Mathematics (BGSMath)
Grav Via 585. 08007 Barcelona, Spain
fontich@ub.edu

Abstract

We consider a map F of class C^r with a fixed point of parabolic type whose differential is not diagonalizable and we study the existence and regularity of the invariant manifolds associated with the fixed point using the parameterization method. Concretely, we show that under suitable conditions on the coefficients of F , there exist invariant curves of class C^r away from the fixed point, and that they are analytic when F is analytic. The differentiability result is obtained as an application of the fiber contraction theorem. We also provide an algorithm to compute an approximation of a parameterization of the invariant curves and a normal form of the restricted dynamics of F on them.

2020 Mathematics Subject Classification: Primary: 37D10; Secondary: 37C25.

Key words and phrases: Parabolic point, invariant manifold, parameterization method.

1 Introduction

Invariant manifolds play a central role in the study of dynamical systems. There is a huge amount of literature devoted to study them in many different settings. In this paper we deal with the invariant manifolds of a type of parabolic fixed points in dimension two.

Parabolic points appear generically in two-parameter families of planar maps or in one-parameter ones in the case of area preserving maps. In particular they appear when a family of maps undergoes a Bogdanov-Takens bifurcation [6, 27].