POLYNOMIAL HAMILTONIAN SYSTEMS OF DEGREE 3 WITH NILPOTENT SADDLES

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Abstract. We provide normal forms and the global phase portraits in the Poincaré disk for all Hamiltonian planar polynomial vector fields of degree 3 symmetric with respect to the $x-$axis having a nilpotent saddle at the origin.

1. Introduction and statement of the results

Let (P, Q) be an analytic map from $\mathbb{R}^2$ into itself. The qualitative theory of ordinary differential equations in the plane provide a qualitative description of the behavior of each orbit (that is, a curve represented by a solution of a differential equation $x' = P$, $y' = Q$) instead of giving explicitly the solutions. More exactly, if $(x(t), y(t))$ is an orbit of that system with maximal interval of definition (α, ω), one of the objectives is to describe its behavior when $t \to \alpha$ or $t \to \omega$, i.e. the α and ω-limit sets of this orbit. To this end, it suffices:

(i) to describe the local phase portraits of singular points;
(ii) to determine the number and the location of limit cycles;
(iii) to determine the α and ω-limit sets of all separatrices of the differential system.

In this paper we will focus on the first one (i) for Hamiltonian systems. We recall that Hamiltonian systems are relevant for many physical studies. Let $H(x, y)$ be a real polynomial in the variables x and y. Then a system of the form

$$x' = H_y \quad y' = -H_x \tag{1}$$

is called a *polynomial Hamiltonian system*. Here the prime denotes derivative with respect to the independent variable t. A system of the form 1 is called a *a polynomial Hamiltonian system of degree d*, if the maximum of the degrees of H_y and H_x is d.

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