HAMILTONIAN NILPOTENT SADDLES OF LINEAR PLUS CUBIC HOMOGENEOUS POLYNOMIAL VECTOR FIELDS

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ABSTRACT. We completely characterize the global phase portraits in the Poincaré disk for all planar Hamiltonian vector fields with linear plus cubic homogeneous terms having a nilpotent saddle at the origin.

1. INTRODUCTION AND STATEMENT OF THE RESULTS

Let (P, Q) be an analytic map from $\mathbb{R}^2$ into itself. The qualitative theory of ordinary differential equations in the plane provide a qualitative description of the behavior of each orbit instead of giving explicitly (or quantitatively) the solutions. In this paper we describe the local phase portraits of singular points for a wide general class of systems being of great interest due to their connection with physical systems.

Quadratic systems having a center at the origin have been widely studied in the last 100 years, and more than 1.000 papers have been published about them (see [10, pages 3 and 4 and 13] for a brief history of the problem of the center in general, and where it includes a list of 300 papers covering this topic.) There are also some partial results for the centers of planar polynomial differential systems of degree larger than two. Recently Colak, Llibre and Valls [3, 4, 5, 6] provided the global phase portraits on the Poincaré disk of all Hamiltonian planar polynomial vector fields having only linear and cubic homogeneous terms which have a linear type center or a nilpotent center at the origin, together with their bifurcation diagrams.

Dulak [8] was the first to detect that centers can pass to saddles through a complex change of variables, see for more details [9], and so it is natural to ask whether such kind of studies can also be done for saddles. It is interesting to observe that despite the fact that the classification of phase portraits of Hamiltonian planar polynomial vector fields having a ceter at the origin have been widely studied very few results exist in the case of saddles. For the case of quadratic systems having an integrable saddle, its phase portraits were provided in [2]. As far as the authors know, for the case in which there exists a nilpotent saddle at the origin and the degree of the system is greater than two no result exists on the classification of the phase portraits. This is the objective pursuit by this paper. This is a huge class of systems with too many parameters and so, in this paper we restrict to classifying the global

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