International Journal of Bifurcation and Chaos, Vol. 30, No. 1 (2020) 2050006 (33 pages)

DOI: 10.1142/S0218127420500066

Global Phase Portraits of $\mathbb{Z}_2$-Symmetric Planar Polynomial Hamiltonian Systems of Degree Three with a Nilpotent Saddle at the Origin

Montserrat Corbera
Facultat de Ciències i Tecnologia. Universitat de Vic, Universitat Central de Catalunya (UVic-UCC), 08500 Vic, Spain
montserrat.corbera@uvic.cat

Claudia Valls
Departamento de Matemàtica, Instituto Superior Técnico, Universidade de Lisboa, 1049-001 Lisboa, Portugal
cvalls@math.ist.utl.pt

Received December 19, 2018; Revised June 23, 2019

We characterize the phase portraits in the Poincaré disk of all planar polynomial Hamiltonian systems of degree three with a nilpotent saddle at the origin and $\mathbb{Z}_2$-symmetric with $(x, y) \mapsto (-x, y)$.

Keywords: Polynomial Hamiltonian systems; nilpotent saddle; phase portrait; Poincaré compactification.

1. Introduction and Statement of the Results

In this paper, we study the global phase portraits of a class of $\mathbb{Z}_2$-symmetric planar polynomial Hamiltonian systems of degree three with a nilpotent saddle at the origin. We recall that a *planar polynomial Hamiltonian system* is a system of the form

$$x' = H_y, \quad y' = -H_x \tag{1}$$

where $H(x, y)$ is a real polynomial of the variables x and y. Here the prime denotes the derivative with respect to the independent variable t. We say that system (1) has *degree* d if the maximum of the degrees of H_y and H_x is d. In this paper, we will focus on the case where $d = 3$.

Let $p \in \mathbb{R}^2$ be a singular point of a polynomial differential system in $\mathbb{R}^2$. Without loss of generality we can assume that p is at the origin of coordinates. We say that p is a *nilpotent saddle* if after a linear change of variables and a rescaling of the time (if necessary) the system can be written in the form

$$x' = y + P(x, y), \quad y' = Q(x, y)$$

where $P(x, y)$ and $Q(x, y)$ are real analytic functions without constant and linear terms, defined in a neighborhood of the origin and satisfying some additional conditions, see Theorem 3.5 in [Dumortier *et al.*, 2006]. In this paper, we will consider the case in which P and Q are polynomials of degree three.

The global phase portraits in the Poincaré disk of all planar polynomial Hamiltonian vector fields with degree three having a nilpotent center at the origin have been provided in several studies (see for instance [Colak *et al.*, 2014; Dias *et al.*, 2018]). However, no global phase portraits in the Poincaré disk are given for planar polynomial Hamiltonian vector fields having a nilpotent saddle at the origin.