



# On Centered Co-circular Central Configurations of the $n$ -Body Problem

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## Abstract

We study the co-circular central configurations of the  $n$ -body problem for which the center of mass and the center of the common circle coincide. In particular, we prove that there are no central configurations of this type with all the masses equal except one. This provides more evidences for the veracity of the conjecture that the regular  $n$ -gon with equal masses is the unique co-circular central configuration of the  $n$ -body problem whose center of mass is the center of the circle. Our result remains valid if we consider power-law potentials.

**Keywords** Co-circular central configurations ·  $n$ -Body problem · Regular  $n$ -gon

**Mathematics Subject Classification** 70F07 · 70F15

## 1 Introduction

A configuration of the Newtonian  $n$ -body problem is *central* if the acceleration vector of each body is a common scalar multiple of its position vector (with respect to the center of mass); i.e. if there exists  $\lambda$  independent of  $i$  such that

$$\ddot{\mathbf{q}}_i = -\lambda (\mathbf{q}_i - \mathbf{c}), \quad i = 1, \dots, n,$$

where  $\mathbf{q}_i$  is the position of the mass  $m_i$  and  $\mathbf{c} = \sum_{i=1}^n m_i \mathbf{q}_i / \sum_{i=1}^n m_i$  is the center of mass of the system; or equivalently, if there exists  $\lambda$  such that

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