On the convex central configurations of the symmetric $(\ell+2)$–body problem

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Abstract For the 4–body problem there is the following conjecture: Given arbitrary positive masses the planar 4–body problem has a unique convex central configuration for each ordering of the masses on its convex hull. Until now this conjecture remains open. Our aim is to prove that this conjecture cannot be extended to the $(\ell+2)$–body problem with $\ell \geq 3$. In particular, we prove that the symmetric $(2n+1)$–body problem with masses $m_1 = \cdots = m_{2n-1} = 1$ and $m_{2n} = m_{2n+1} = m$ sufficiently small has at least two classes of convex central configuration when $n = 2$, five when $n = 3$, and four when $n = 4$. We conjecture that the $(2n+1)$-body problem has at least n classes of convex central configurations for $n > 4$ and we give some numerical evidences that the conjecture can be true. We also prove that the symmetric $(2n+2)$–body problem with masses $m_1 = \cdots = m_{2n} = 1$ and $m_{2n+1} = m_{2n+2} = m$ sufficiently small has at least three classes of convex central configuration when $n = 3$, two when $n = 4$, and three when $n = 5$. We also conjecture

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