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ORIGINAL ARTICLE

Classifying four-body convex central configurations

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Abstract

We classify the full set of convex central configurations in the Newtonian planar four-body problem. Particular attention is given to configurations possessing some type of symmetry or defining geometric property. Special cases considered include kite, trapezoidal, co-circular, equidiagonal, orthodiagonal, and bisecting-diagonal configurations. Good coordinates for describing the set are established. We use them to prove that the set of four-body convex central configurations with positive masses is three-dimensional, a graph over a domain D that is the union of elementary regions in $\mathbb{R}^{+^3}$.

Keywords Central configuration · n-Body problem · Convex central configurations

1 Introduction

The study of central configurations in the Newtonian n-body problem is an active subfield of Celestial Mechanics. A configuration is *central* if the gravitational force on each body is a common scalar multiple of its position vector with respect to the center of mass. Perhaps the most well-known example is the equilateral triangle solution of Lagrange, discovered in 1772, consisting of three bodies of arbitrary mass located at the vertices of an equilateral triangle (Lagrange 1772). Released from rest, any central configuration will collapse homothetically toward its center of mass, ending in total collision. In fact, any solution of the n-body problem containing a collision must have its colliding bodies asymptotically approaching a central configuration (Saari 2005). On the other hand, given the appropriate initial velocities, a planar

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