

ASYMPTOTIC DYNAMICS OF A DIFFERENCE EQUATION WITH A PARABOLIC EQUILIBRIUM

B. COLL[†], A. GASULL[‡] AND R. PROHENS^{*}

ABSTRACT. The aim of this work is the study of the asymptotic dynamical behaviour, of solutions that approach parabolic fixed points in difference equations. In one dimensional difference equations, we present the asymptotic development for positive solutions tending to the fixed point. For higher dimensions, through the study of two families of difference equations in the two and three dimensional case, we take a look at the asymptotic dynamic behaviour. To show the existence of solutions we rely on the parametrization method.

1. INTRODUCTION AND MAIN RESULTS

In studying difference equations, one interesting problem is to know the asymptotic dynamical behaviour, of the positive solutions that approach equilibrium points. This question naturally arises, for instance, when describing applications in biological or economical systems. See [8, 17, 20, 21, 29, 31], for instance. In particular, how populations evolve to equilibrium states and, more concretely, if they tend to rest points in the same or different asymptotically way is of great interest in mathematical modeling.

This problem appears not only in modeling real-life processes but also in Celestial Mechanics ([14, 22, 16, 26, 28]), in complex analytical dynamics ([23]) and in many more others fields where the subject is concerned with the study of invariant submanifolds of fixed parabolic points of maps.

Related to determine the complete asymptotic expansion of solutions tending to fixed points in difference equations are next three examples appearing in the literature, in which this issue is unsolved.

Key words and phrases. Difference equations, parabolic equilibrium, asymptotic development

2010 Mathematics Subject Classification: 39A10, 41A60, 39A30, 37C75 .

The second author is partially supported by Spanish MCYT/FEDER grant number MTM2016-77278-P and Gov. Catalunya grant number 2017SGR 1617; the first and third authors are partially supported by Spanish and European Regional Development Funds (ERDF, FEDER) MICINN MTM2017-83568-P grants. The third author is also partially supported by MICINN MTM2014-54275-P Spanish grant.