

Pointwise periodic maps with quantized first integrals

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Abstract

In a series of papers, Chang, Cheng and Wang studied the periodic behavior of some piecewise linear maps in the plane. These examples are valuable under the light of the classical result of Montgomery about periodic homeomorphisms, since they are pointwise periodic but not periodic. We revisit them from the point of view of their properties as integrable systems. We describe their global dynamics in terms of the dynamics induced by the maps on the level sets of certain first integrals that we also find. We believe that some of the features that the first integrals exhibit are interesting by themselves, for instance the set of values of the integrals are discrete. Furthermore, the level sets are bounded sets whose interior is formed by a finite number of some prescribed tiles of certain regular tessellations. The existence of these quantized integrals is quite novel in the context of discrete dynamic systems theory.

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