Phase portraits of random planar homogeneous vector fields*

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November 25, 2019

Abstract

We study the phase portraits with positive probability of random planar homogeneous vector fields of degree n. In particular, for $n = 1, 2, 3$, we give a complete solution of the problem and, moreover, either we give the exact value of each probability or we estimate it by using the Monte Carlo method. It is remarkable that all but two of these phase portraits are characterized by their index at the origin and by their number of invariant straight lines through it.

Mathematics Subject Classification 2010: 37H10, 34F05.

Keywords: Ordinary differential equations with random coefficients; planar homogeneous vector fields; index; phase portraits.

1 Introduction and main results

The systems of ordinary differential equations, or equivalently vector fields, are ubiquitous tools in the mathematical modeling of physical phenomenon. When studying parametric

*The authors are supported by Ministry of Economy, Industry and Competitiveness–State Research Agency of the Spanish Government through grants MTM2016-77278-P (MINECO/AEI/FEDER, UE, first and second authors) and DPI2016-77407-P (MINECO/AEI/FEDER, UE, third author). The first and second authors are also supported by the grant 2017-SGR-1617 from AGAUR, Generalitat de Catalunya. The third author acknowledges the group's research recognition 2017-SGR-388 from AGAUR, Generalitat de Catalunya.