

Stability index of linear random dynamical systems*

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Abstract

Given a homogeneous linear discrete or continuous dynamical system, its stability index is given by the dimension of the stable manifold of the zero solution. In particular, for the n dimensional case, the zero solution is globally asymptotically stable if and only if this stability index is n . Fixed n , let $p_k, k = 0, 1, \dots, n$, denote the probabilities that the random variable that assigns to each linear random dynamical system its stability index takes the value k . In this paper we obtain either the exact values p_k , or their estimations by combining the Monte Carlo method with a least square approach that uses some affine relations among the values $p_k, k = 0, 1, \dots, n$. The particular case of n -order homogeneous linear random differential or difference equations is also studied in detail.

Mathematics Subject Classification 2010: 37H10, 34F05, 39A25, 37C75.

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