THE LIMIT CYCLES OF THE HIGGINS-SELKOV SYSTEMS

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Abstract. In this paper we investigate the problem of limit cycles for general Higgins-Selkov systems with degree $n+1$. In particular, we first prove the uniqueness of limit cycles for a general Liénard system which allows for discontinuity. Then, by changing the Higgins-Selkov systems into Liénard systems, theorems and some techniques for Liénard systems can be applied. After we prove the nonexistence of limit cycles if the bifurcation parameter is outside an open interval. Finally we complete the analysis of limit cycles for the Higgins-Selkov systems showing its uniqueness.

1. Introduction and main results

In the qualitative theory of planar polynomial differential systems it is well known how difficult is to study the famous Hilbert's 16th problem, see [8, 10, 16]. Up to now there are seldom works having solved the problem of exact number of limit cycles for polynomial differential systems.

The most important physiological function of carbohydrates is to provide energy for organisms' life activities. Glucose catabolism is the main way for organisms to obtain energy. There are three main pathways for the oxidative decomposition of glucose in organisms. Among them, the anaerobic oxidation of glucose is called glycolysis. We consider the following polynomial differential system of arbitrary degree

$$\begin{aligned} \dot{x} &= 1 - xy^n, \\ \dot{y} &= ay(-1 + xy^{n-1}) \end{aligned} \tag{1}$$

which was proposed first by Higgins [7] and modified further by Selkov [13] for studying the biological nonlinear glycolytic oscillations, and was called the Higgins-Selkov system. Here n is a positive integer and a is a real parameter. Artés, Llibre and Valls in [1] characterized the global dynamics described in the Poincaré disc for system (1) as $n = 2$ and $a \in \mathbb{R}\backslash(1, 3)$. Moreover there are two conjectures stated in [1] on the the number of limit cycles of systems (1) when $a \in (1, 3)$. After Chen and Tang in [5] proved these conjectures which complete the global phase portraits of system (1) when $n = 2$.

Recently Brechmann and Rendall in [3] researched the uniqueness of limit cycles for system (1) and additionally proved that no limit cycles exist when $a \in (0, 1/(1-n))$. Llibre and Mousavi [11] classified the phase portraits of system (1) for $n = 3, 4, 5, 6$ in the Poincaré disc for all the values of the parameter a and determined in function of the parameter a the regions of the phase space with biological meaning.

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