PHASE PORTRAITS AND BIFURCATION DIAGRAM OF THE GRAY-SCOTT MODEL

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ABSTRACT. We give a complete classification of the phase portraits in the Poincaré disk for the cubic polynomial systems

$$\dot{x} = 1 - x - axy^2, \qquad \dot{y} = -by + axy^2,$$

in $\mathbb{R}^2$ according with the values of its two parameters a and b. These differential systems correspond to the Gray-Scott model. Moreover we provide the bifurcation diagram in the parameter plane (a, b) of these systems.

The Gray-Scott model for studying the autocatalysis has been analyzed by several authors these recent years. The original partial differential equations can be simplified to ordinary differential equations, more precisely to a family cubic polynomial differential systems depending on two parameters a and b. Here we characterize the global dynamics of these cubic systems, taking into account their behavior near the infinity using the Poincaré compactification. Moreover we provide the bifurcation diagram of the phase portraits in the parameter plane (a, b).

1. Introduction and statement of the main results

The Gray-Scott model [13, 18] is a cubic autocatalysis system that exhibits many interesting patterns, see for instance the papers [3, 10, 13, 16, 18] and the references quoted there. This model has been studied with slightly distinct differential equations, here we consider the model given by the differential equations

$$\begin{aligned} \frac{\partial U}{\partial t} &= 1 - U - aUV^2 + D_U \Delta U, \\ \frac{\partial V}{\partial t} &= -bV + aUV^2 + D_V \Delta V, \end{aligned} \tag{1}$$

where $U = U(u, v, t)$ and $V = V(u, v, t)$ are the concentrations of an inhibitor and an activator, $(u, v) \in \mathbb{R}^2$, a and b are positive constants, $D_U > D_V$ are the diffusivities, Δ is the Laplace operator, and t is the time. For more details on the Gray-Scott model see the previous mentioned references.

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