MELNIKOV FUNCTIONS OF ARBITRARY ORDER FOR PIECEWISE SMOOTH DIFFERENTIAL SYSTEMS IN $\mathbb{R}^n$ AND APPLICATIONS

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Abstract. In this paper we develop an arbitrary order Melnikov function to study limit cycles bifurcating from a periodic submanifold for autonomous piecewise smooth differential systems in $\mathbb{R}^n$ with two zones separated by a hyperplane. This result not only extends some of the known results on the Melnikov theory in dimension and order but also compensates for some defects of the averaging theory in studying the limit cycle bifurcation of autonomous systems from a periodic submanifold. To demonstrate the application of our theoretical result and its superiority for some systems to the existing averaging theory, we study the maximum number of limit cycles bifurcating from an n-dimensional periodic submanifold caused by non-smooth centers of the fold-fold type, providing an upper bound for any order piecewise polynomial perturbations of degree m. Concerning the planar case of the unperturbed system, a piecewise Hamiltonian system, we obtain a better upper bound for piecewise polynomial Hamiltonian perturbations up to order two. The realizability of these upper bounds is also discussed.

1. Introduction and statement of the main results

Qualitative theory of piecewise smooth (PWS) differential systems has become one of the most booming research objects of ordinary differential equations in recent years. With the help of such systems, we can better model and analyze discontinuous phenomena in nature, such as the switching of circuit systems, the impact of mechanical devices, the activity of neurons in the central nervous system, the vibration of oscillators with dry friction, see [3,12,25,44]. On the contrary, an in-depth understanding of these discontinuous phenomena also has inspired the investigation of PWS systems.

Consider the n-dimensional PWS system

$$\dot{\mathbf{x}} = \begin{cases} \mathbf{f}^+(\mathbf{x};\varepsilon) & \text{if} \quad \mathbf{x} \in \Sigma^+, \\ \mathbf{f}^-(\mathbf{x};\varepsilon) & \text{if} \quad \mathbf{x} \in \Sigma^-, \end{cases} \tag{1}$$

where $\mathbf{x} \in \mathbb{R}^n$, $n \geq 2$, $\varepsilon \in \mathbb{R}$ is a perturbation parameter, $\mathbf{f}^\pm : \mathbb{R}^n \times \mathbb{R} \to \mathbb{R}^n$ are analytic functions,

$$\Sigma^+ = \{\mathbf{x} \in \mathbb{R}^n : \pi\mathbf{x} > 0\}, \qquad \Sigma^- = \{\mathbf{x} \in \mathbb{R}^n : \pi\mathbf{x} < 0\}$$

are two zones separated by the hyperplane $\Sigma = \{\mathbf{x} \in \mathbb{R}^n : \pi\mathbf{x} = 0\}$, usually called *discontinuity boundary* or *switching boundary* [3]. We denote by $\pi : \mathbb{R}^n \to \mathbb{R}$ the projection onto the first coordinate and by $\pi^\perp : \mathbb{R}^n \to \mathbb{R}^{n-1}$ the projection onto the last $n-1$ ones. For system (1)

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