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Simultaneous occurrence of sliding and crossing limit cycles in piecewise linear planar vector fields

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ABSTRACT

In the present study, we consider planar piecewise linear vector fields with two zones separated by the straight line $x = 0$. Our goal is to study the existence of simultaneous crossing and sliding limit cycles for such a class of vector fields. First, we provide a canonical form for these systems assuming that each linear system has centre, a real one for $y < 0$ and a virtual one for $y > 0$, and such that the real centre is a global centre. Then, working with a first-order piecewise linear perturbation we obtain piecewise linear differential systems with three crossing limit cycles. Second, we see that a sliding cycle can be detected after a second-order piecewise linear perturbation. Finally, imposing the existence of a sliding limit cycle we prove that only one adittional crossing limit cycle can appear. Furthermore, we also characterize the stability of the higher amplitude limit cycle and of the infinity. The main techniques used in our proofs are the Melnikov method, the Extended Chebyshev systems with positive accuracy, and the Bendixson transformation.

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1. Introduction and statement of the main results

For a given differential system a limit cycle is a periodic orbit isolated in the set of all periodic orbits of the system. One of the main problems of the qualitative theory of planar differential systems is determining the existence of limit cycles. A *centre* is a singular point p that possesses a neighborhood U such that $U \setminus \{p\}$ is filled by periodic solutions. A classical way to produce and study limit cycles is by perturbing the periodic solutions of a centre. This problem has been studied intensively for continuous planar differential systems, see for instance, [5] and the references therein.

In this paper, we are concerned in limit cycles bifurcating from a centre of discontinuous piecewise linear differential systems with two zones separated by the straight line $x = 0$, when the centre is perturbed inside the class of all discontinuous piecewise linear differential systems with two zones separated by $x = 0$.

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